

performance, at some stage, our portfolio earns nearly 1.5%. Now, let's think as a(n) (algorithmic) trader. If we open this portfolio with a triple leverage, we might exceed 5% profit easily at the same underlying level of risk! Of course, the leverage would cause added volatility to portfolio performance but, as they say, *if there is no risk there is no reward*.

## Portfolio Optimization with Risk under Control

Since the beginning we have been mentioning the possibility of the inclusion of the risk-free asset. Based on the theoretical works of Sharpe in 1964, Lintner in 1965 and Tobin in 1958, the importance of the risk-free asset in the portfolio has been proved to equip us with a better control over risk. As we will also see in a moment, it gives us a balanced approach to our investment. We can split the budget into fractions of our capital designated for an investment in the risk-free option (e.g. the savings account in the bank) while the rest will be delegated to other assets of diversified risk levels.

It was shown that for any portfolio with the risk-free component, the expected return is:

$$R_P = mw^T + (1 - \mathbf{1}w^T)r_f \quad \text{at} \quad \sigma_P = wM_2w^T$$

Given that, we can formulate the minimum variance portfolio optimization problem in the following framework:

$$\begin{aligned} \min_w \quad & wM_2w^T \\ \text{s.t.} \quad & mw^T + r_f(1 - \mathbf{1}w^T) = r_{\text{target}} \end{aligned}$$

It occurs that all minimum variance portfolios are a combination of the risk-free asset and a given risky portfolio. The latter is often called the **tangent portfolio** and has been shown that it must contain of all assets available to investors (held in quantity to its market value relative to the total market value of all assets). That makes the second name of the tangent portfolio, namely, the **market portfolio**.

Luckily for solvers, the objective function of the form:

$$\begin{aligned} \max_w \quad & \frac{mw^T - r_f}{\sqrt{wM_2w^T}} \\ \text{s.t.} \quad & \mathbf{1}w^T = 1 \end{aligned}$$

called the **Sharpe ratio** corresponds to the market portfolio directly. The risk-free asset is connected with the tangent portfolio by the straight line, as deduced from the formulas abovementioned, therefore provides an investor with a good blend of risk-controlled portfolios. Let's see how we can play with these tools in Matlab for the needs of the better **risk management** in practice.

Matlab's Financial Toolbox has been enriched by a set of useful functions designed to help investors and algorithmic traders with finding optimal solutions. Defining the complex optimization problems with complicated non-linear constraints is troublesome in many cases. Here, we touch these problems in a gentle way simply to provide a good ground for the understanding of usefulness of Matlab in this domain. The 2nd Edition of the e-book will address much wider spectrum of similar queries.

Below, we continue with our recent 30-asset portfolio of Dow Jones Index components and, eventually, we include  $r_f = 4.4\%$  p.a. risk-free element in the game.

With the basic setup as at the beginning, we roll:

```
% Construction of the 30-asset portfolio
fname=['portfolio2.lst']
ndays=365;
dt='daily';
lday=datetime('2014-02-13')
[X,Y,R,AssetList] = FetchQuandlForPortfolio(fname,ndays,lday,dt);

CashRet=0.044/252; % riskless asset (e.g. savings account earning 4.4% pa)
CashRsk=0;        % riskless risk

% Default portfolio constructor based on 30 return-series, R matrix
p=Portfolio('RiskFreeRate',CashRet);
p=p.setAssetList(AssetList);
p=p.estimateAssetMoments(R);
p=p.setDefaultConstraints;
[pAssetMean, pAssetCovar] = p.getAssetMoments;
pfrontier = p.estimateFrontier(1000);
[prsk, pret] = p.estimatePortMoments(pfrontier);
```

Now, for the needs of building the extension of the efficient frontier, the **market line**, connecting the risk-free asset with the tangent portfolio (not yet defined in the code), we proceed with:

```
q = p.setBudget(0,1); % ***
estq = q.estimateFrontier(1000);
[qrsk, qret] = q.estimatePortMoments(estq);
```

what communicates to the solver that we now allow the total capital (setBudget function) to be freely split between the efficient frontier's portfolios and risk-free asset (weights given between 0.0 and 1.0). Variables of qret and qrsk will hold the vectors of returns and risk corresponding to the market line.

Now, we supplement the picture with the best estimation of the tangent portfolio:

```
p = p.setBudget(1,1); % allocation of 100% of our capital into risky assets (stocks)
p = p.setInitPort(0);
estt = p.estimateMaxSharpeRatio;
[trsk, tret] = p.estimatePortMoments(estt);
```

by going back to the portfolio constructor of p. Making use of the dedicated function of estimateMaxSharpeRatio we pass the object of estt into the function which optimises our 30-asset portfolio targeting solution for most optimal portfolio weights (defining our tangent portfolio). As usual we plot the battlefield with:

```
figure(7);
clf;
plotportfolio('Tangent Portfolio without Risk-Free Asset',dt,'other',20, ...
    {'line', prsk, pret}, ...
    {'line', qrsk, qret, [], [], 1}, ...
    {'scatter', trsk, tret, {'Tangent'}}, ...
    {'scatter', CashRsk, CashRet, {'Cash'}}, ...
    {'scatter', sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, '.r'});
ylim([-0.02 0.06]); xlim([0 0.1]);
```

and list the components of the tangent portfolio found by solver:

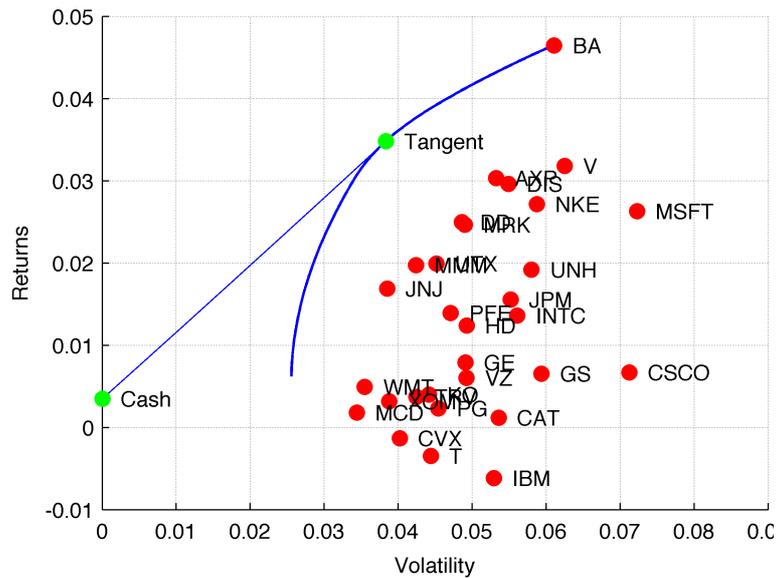
```
W=dataset({100*estt(estt > 0), 'Weight'}, 'obsnames', AssetList(estt > 0));
fprintf('Optimised Portfolio\n');
disp(W);
instocks=sum(100*estt(estt>0)); incash=100-instocks;
fprintf('In stocks: %5.2f\nIn cash: %5.2f\n\n', instocks, incash);
```

The aforementioned line of coding delivers us the tangent portfolio solution:

Optimised Portfolio	
	Weight
AXP	6.2513
BA	39.912
DIS	4.6607
MRK	24.701
MSFT	10.191
NKE	5.8806
V	8.4046

In stocks: 100.00  
In cash: 0.00

marked and labelled in the risk-return diagram as follows:



It promises:

```
>> 100*((tret+1)^20-1) % percent
```

```
ans =  
    3.4824
```

profit at 3.83% of risk over next 20 business days. Please note that the tangent portfolio is the best optimised trade-off between risk and return for the underlying portfolio. Now, these numbers do not take into account the risk-free asset. Our intuition whispers us that if we start moving towards lower volatilities, we should leave the curved efficient frontier and step onto the road leading us eventually to the duty-free zone. That is where our cash in the bank sleeps peacefully.

Therefore, if we replace our code from the previous page starting from the line containing the marker `***` in the comment with the following block of code:

```
q = p.setBudget(0,1);  
estq = q.estimateFrontier(1000);  
[qrsk, qret] = q.estimatePortMoments(estq);  
  
p = p.setBudget(0,1);  
p = p.setInitPort(0);  
estt = p.estimateMaxSharpeRatio;  
[trsk, tret] = p.estimatePortMoments(estt);  
  
figure(7);  
clf;
```